

Math 141  
Joseph C Foster  
Summer 2017  
Midterm 2

Name: \_\_\_\_\_  
June 1st, 2017  
Time Limit: 55 minutes

---

This exam contains 10 pages (including this cover page) and 13 questions.  
The total number of points is 100. You have 55 minutes to complete the exam.

Read each question carefully. When specified, you must show **all necessary** work to receive full credit.

No calculator/phone/smartwatch allowed under any circumstances. Place these items in your bag, out of reach. Cheating of any kind will not be tolerated and will result in a grade of zero.

Question	Marks	Score	Question	Marks	Score
1	5		8	8	
2	5		9	18	
3	5		10	12	
4	5		11	12	
5	5		12	12	
6	5		13	12	
7	8		Total	100	

---

1. (5 marks) True or False: If  $f(x)$  is continuous on an open interval  $(a, b)$ , then  $f$  attains both an absolute maximum and an absolute minimum in  $(a, b)$ .
- A. True B. False
2. (5 marks) Fill in the blank: Let  $f(x)$  be continuous over the interval  $[a, b]$  and differentiable at every point of its interior  $(a, b)$ . If  $f(a) = f(b)$  then there is at least one number  $c \in (a, b)$  at which \_\_\_\_\_.
3. (5 marks) Fill in the blank: Let  $f(x)$  be a differentiable function. Let  $c \in \mathbb{R}$  be a point such that  $f'(c) = 0$ . Then;
- If  $f''(c) < 0$ ,  $f(c)$  is a local \_\_\_\_\_
  - If  $f''(c) > 0$ ,  $f(c)$  is a local \_\_\_\_\_
  - If  $f''(c) = 0$ ,  $f(c)$  could be either.

For questions 4-6, choose the best answer. There is only one correct answer but you may choose up to *two*. If you choose two and one of the answers is correct, you will receive 2 marks.

4. (5 marks) What is the derivative of  $f(x) = \sin^{-1}(x)$ ?

A.  $\frac{1}{\sqrt{1-x^2}}$

C.  $\frac{1}{1+x^2}$

B.  $-\frac{1}{\sqrt{1-x^2}}$

D.  $-\frac{1}{1+x^2}$

5. (5 marks) What is the derivative of  $f(x) = \tan^{-1}(x)$ ?

A.  $\frac{1}{\sqrt{1-x^2}}$

C.  $\frac{1}{1+x^2}$

B.  $-\frac{1}{\sqrt{1-x^2}}$

D.  $-\frac{1}{1+x^2}$

6. (5 marks) Which of the following is the tangent line to the function  $f(x) = x^2 + 1$  at the point  $(2, 5)$ ?

A.  $y = 2x + 1$

C.  $y = 4x - 3$

B.  $y = 3x - 1$

D.  $y = 5x - 5$

In questions 7-17 you must show **all necessary work** to receive full credit. Unless specified, you do not need to simplify your final answer.

7. (8 marks) Prove that the function  $f(x) = x^5 + x^3 - 1$  has *exactly* one real root.

8. (8 marks) Evaluate  $\lim_{x \rightarrow \infty} (e^x + x)^{1/x}$

9. Let  $f(x) = \frac{3(x^2 + 1)}{x^2 - 9}$ . Then  $f'(x) = -\frac{60x}{(x^2 - 9)^2}$  and  $f''(x) = \frac{180(x^2 + 3)}{(x^2 - 9)^3}$ . When asked for points, include *both* the  $x$  and  $y$  coordinate.

(a) i. (1 mark) Does the curve intercept the  $x$ -axis? If so, give the point(s) of interception. If not, say why.

ii. (1 mark) Does the curve intercept the  $y$ -axis? If so, give the point(s) of interception. If not, say why.

(b) i. (1 mark) Does the curve have any vertical asymptotes? If so what are they?

ii. (1 mark) Calculate  $\lim_{x \rightarrow \infty} f(x)$ . Does the curve have an asymptote at positive infinity? If so, what is it?

iii. (1 mark) Calculate  $\lim_{x \rightarrow -\infty} f(x)$ . Does the curve have an asymptote at negative infinity? If so, what is it?

(c) i. (1 mark) For what values of  $x$  is  $f(x)$  positive?

ii. (1 mark) For what values of  $x$  is  $f(x)$  negative?

(d) i. (2 marks) What are the critical points of  $f(x)$ ?

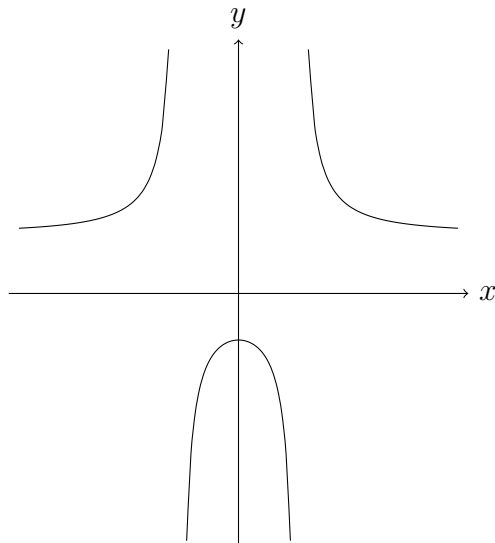
ii. (1 mark) For what values of  $x$  is  $f(x)$  increasing?

iii. (1 mark) For what values of  $x$  is  $f(x)$  decreasing?

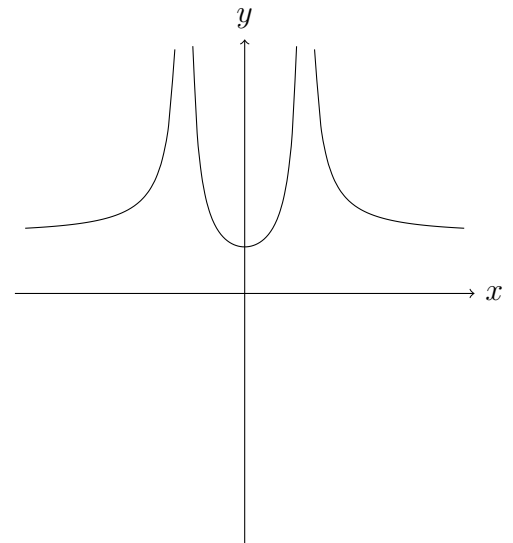
(e) (3 marks) Classify the critical point(s) you found above as either maximum or minimum.

(f) (4 marks) Which of the curves below represent  $f(x)$ . Choose only *one*. Label the points found above on your choice.

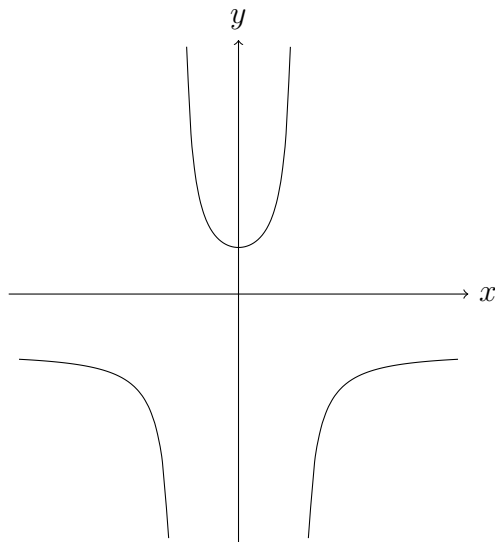
A.



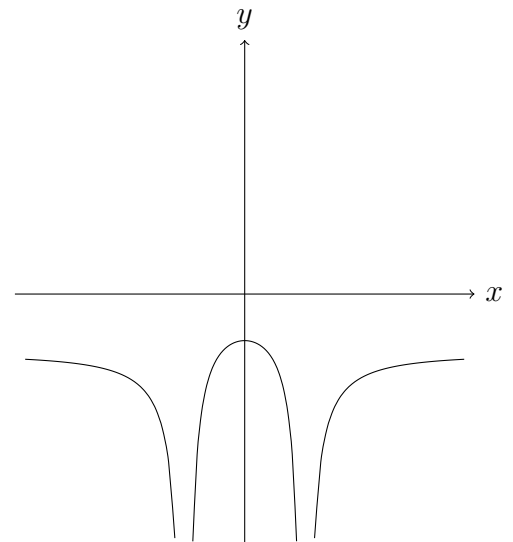
C.



B.



D.



Of questions 10 – 13, you must answer **three questions**. If you answer more than three then only **your best three scores will be counted**.

10. (12 marks) Coffee is draining from a conical filter, of height and diameter 6cm, into a cylindrical coffee pot, also of diameter 6cm, at a rate of  $10\text{cm}^3/\text{min}$ . Calculate how fast the levels in both the pot and the cone are changing when the coffee in the cone is 5cm deep.

11. (12 marks) Assume that sand allowed to pour onto a level surface will form a pile in the shape of a cone, with height equal to the diameter of its base. If sand is poured at  $2\text{m}^3/\text{sec}$ , how fast is the height of the pile changing when its base is 8m in diameter.



12. (12 marks) We want to construct a box whose base length is 3 times the base width. The material used to build the top and bottom cost \$10 per square foot and the material used to build the sides cost \$6 per square foot. If the box must have a volume of  $50\text{ft}^3$ , determine the dimensions that will minimise the cost to build the box.

13. (12 marks) A piece of pipe is being carried down a hallway that is 10 feet wide. At the end of the hallway there is a right-angled turn and the hallway narrows to 8 feet wide. What is the longest pipe that can be carried (always keeping it horizontal) around the turn in the hallway?